

**How do fractions compare?**

School grade: K7/K8

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### **How do positive fractions compare?**

* If two positive fractions have the same denominator, then the fraction with the larger numerator is greater than the other: 2/7 < 6/7. Why? 7 parts of a larger number, 6, is always greater than 7 parts of a smaller number, 2;
* If two positive fractions have the same numerator, the fraction with the larger denominator is smaller than the other: 5/9 < 5/7. Why? When we divide the same quantity, 5, into fewer parts, 7, the result is greater than when we divide it into more parts, 9;

In the case of two positive fractions with different numerators and denominators::

* any positive subunit fraction (which is less than 1) is less than any equiunit fraction (which is equal to 1), which in turn is less than any superunit fraction (which is greater than 1):

3/7 < 1 < 5/2

* if the fractions are both subunit or superunit, they are brought to the same denominator first, the fraction with the larger numerator is greater than the other:

8/9 ? 5/7

(8 × 7) / (9 × 7) ? (5 × 9) / (7 × 9)

56/63 > 45/63

8/9 > 5/7

### **How do negative fractions compare?**

* If two negative fractions have the same denominator, then the fraction with the larger numerator is less than the other: - 2/7 > - 6/7
* If two negative fractions have the same numerator, the fraction with the larger denominator is greater than the other: - 5/9 > - 5/7

In the case of two negative fractions with different numerators and denominators:

* any negative subunit fraction (which is greater than -1) is greater than any negative equiunit fraction (which is equal to -1), which in turn is greater than any negative superunit fraction (which is less than -1):

- 3/7 > -1 > - 5/2

if the fractions are both subunit or superunit, they are brought to the same denominator first, the fraction with the larger numerator is less than the other:

- 8/9 ? - 5/7

- (8 × 7) / (9 × 7) ? - (5 × 9) / (7 × 9)

- 56/63 < - 45/63

- 8/9 < - 5/7

**Amplification and simplification to equivalent fractions, examples**

### **Amplifying and simplifying a fraction**

If the numerator and denominator of a fraction A are multiples of the numerator and denominator respectively of another fraction, B, we say that the fraction A was obtained by multiplying the fraction B.

For example:

8/9 = (8 × 5) / (9 × 5) = 40/45

In this case we say that the fraction 40/45 was obtained by multiplying the fraction 8/9 - more precisely, by multiplying both the numerator and the denominator by the number 5.

Multiplying a fraction means multiplying both the numerator and the denominator of the fraction by the same non-zero number, this operation generating an equivalent fraction::

a/b = (a × c) / (b × c)

The inverse operation of amplification is called simplification.

Simplifying means dividing both the numerator and the denominator of the fraction by the same non-zero number, this operation generating an equivalent fraction

a/b = (a : c) / (b : c)

Operation:

2/7 = (2 × 3) / (7 × 3) = 6/21

represents, from left to right, an amplification, and from right to left a simplification.

### **What fractions can be simplified? Irreducible fractions.**

An ordinary fraction in which the numerator and denominator are coprime numbers (their only common factor is 1) is called an irreducible fraction and cannot be simplified.

The fraction 4/16 is not irreducible and can be simplified, since both 4 and 16 are divisible by 4.

In contrast, the fraction 4/5 is irreducible and cannot be simplified, since the only common factor of 4 and 5 is 1.

In conclusion, any fraction in which the denominator and numerator contain common factors other than 1 can be simplified, that is, the numbers are not coprime.

### **Why do we simplify a fraction?**

Simplifying fractions is indicated, because this operation reduces both the value of the denominator and the numerator, making the calculations in which the respective fractions will be used easier.

**Learn how to simplify fractions to equivalent forms. Irreducible fractions. Common prime factors. Greatest Common Divisor, CMMDC. Examples**

***Simplifying fractions. Equivalent fractions***

***Let's learn by an example, let's simplify the fraction: 12/16***

The numerator of the fraction. The number above the fraction line, 12, is called the numerator of the fraction.

The denominator of the fraction. The number below the fraction line, 16, is called the denominator of the fraction.

The value of the fraction. The fraction 12/16 tells us how many equal parts the number above the fraction line is divided into: 12 is divided into 16 equal parts. Thus, the value of the fraction is calculated as:

12 : 16 = 0,75

We notice that the two numbers, the numerator and the denominator, are evenly divisible by 2, so we divide them by the same number, 2:

12/16 = (12 : 2)/(16 : 2) = 6/8

The value of the fraction 6/8 is calculated as:

6 : 8 = 0,75

We note that the value of the fraction 6/8 is equal to the value of the fraction 12/16, i.e. 0.75

**Simplified fraction.** Equivalent fraction. The fraction obtained, 6/8, is called a fraction equivalent to the original fraction 12/16, that is, it represents the same value, the same proportion of the whole, and was obtained from the original fraction by simplification: both the numerator and the denominator were divided by the number 2.

## **How do two fractions compare?**

### **1. Fractions of different sign**

Any positive fraction is greater than any negative fraction:

ex: 4/25 > - 19/2

### **2. One fraction is subunit, another is supraunit**

Any superunit positive fraction is greater than any equiunit positive fraction, which in turn is greater than any subunit positive fraction::

ex: 44/25 > 1 > 19/200

Any superunit negative fraction is less than any equiunit negative fraction, which in turn is less than any subunit negative fraction:

ex: - 44/25 < -1 < - 19/200

### **3. Fractions with equal numerators but also with equal denominators**

The fractions are equal:

ex: 89/50 = 89/50

### **4. Fractions with different numerators but equal denominators**

**Positive fractions:** the numerators are compared, the larger fraction is the one with the larger numerator:

ex: 74/25 > 49/25

**Negative fractions**: the numerators are compared, the larger fraction is the one with the smaller numerator

ex: - 19/25 < - 17/25

### **5. Fractions with different denominators but equal numerators**

**Positive fractions**: denominators are compared, the larger fraction is the one with the smaller denominator:

ex: 24/25 > 24/26

**Negative fractions**: denominators are compared, the larger fraction is the one with the larger denominator:

ex: - 17/25 < - 17/29

### **6. Fractions with different denominators and numerators**

In order to compare them, the fractions must first be brought to the same denominator (or if it's easier, brought to the same numerator).

#### 1) If necessary, simplify the fractions to their simplest, irreducible equivalent form.

* Factor the numerator and denominator of each fraction into prime factors, specifically as a product of prime factors in exponents.
* Computes the greatest common divisor, CMMDC, of ​​the numerator and denominator of each separate fraction: multiply their common prime factors, uniquely, to the lowest powers.
* We will calculate one CMMDC for each individual fraction.
* Each calculated CMMDC will be used to divide both the numerator and denominator of each fraction to simplify that fraction.
* Divide both the numerator and denominator of each fraction by their greatest common divisor, CMMDC.
* At this point the fractions are simplified to the simplest, irreducible equivalent form.
* By simplifying, the value of the fraction is not changed, but only an equivalent fraction is obtained.2) Calculează cel mai mic multiplu comun, CMMMC, al tuturor numitorilor fracțiilor.
* CMMMC will be the new denominator of the compared equivalent fractions.
* Decompose the denominators of fractions into prime factors, as products of prime factors, in exponent writing.
* To calculate the CMMMC multiply all the prime factors that appear in the denominator decomposition, uniquely, to the highest powers.3) Compară numărătorii noilor fracții echivalente.
* At this point, the fractions are brought to the same denominator, so it simply remains to compare the numerators of the new fractions.
* The larger fraction is the one with the larger numerator, if the fractions are positive.
* If they are negative, the larger fraction is the one with the smaller numerator.

### ***Example*, compare two subunit fractions of the same sign, with different denominators and numerators, with explanations: 16/24 vs. 45/75**

#### 1) We simplify the fractions to their simplest, irreducible equivalent form:

Fraction 16/24:

* Decompose the numerator and denominator into the product of prime factors in exponential notation:
* 16 = 24;
* 24 = 23 × 3;
* Calculates the greatest common divisor, CMMDC, of ​​the numerator and denominator of the fraction, multiply all their common prime factors, to the lowest Powers:
* CMMDC (16; 24) = CMMDC (24; 23 × 3) = 23;
* Divide both numerator and denominator by the greatest common divisor, CMMDC:
* 16/24 = 24 / (23× 3) = (24 : 23) / ((23× 3) : 23) = 2/3.

Fraction 45/75:

* Decompose the numerator and denominator into the product of prime factors in exponential notation:
* 45 = 32 × 5;
* 75 = 3 × 52;
* Calculate the greatest common divisor, CMMDC, of ​​the numerator and denominator of the fraction, multiply all their common prime factors, to the lowest powers:
* CMMDC (45; 75) = CMMDC (32 × 5; 3 × 52) = 3 × 5;
* Divide both numerator and denominator by the greatest common divisor, CMMDC:
* 45/75 = (32 × 5) / (3 × 52) = ((32 × 5) : (3 × 5)) / ((3 × 52) : (3 × 5)) = 3/5.

Simplified fractions are:

16/24 = 2/3;

45/75 = 3/5.

Simplified fractions are fractions equivalent to the original fractions, each having the same value as the original fraction.

16/24 ≈ 0,67; 2/3 ≈ 0,67;

45/75 = 0,6; 3/5 = 0,6;

#### 2) Compare the numerators of equivalent fractions.

Since the fractions now have the same denominator, all that remains is to compare their numerators.

* 10 > 9 => 10/15 > 9/15 => 16/24 > 45/75.

**Learn how to sort fractions with different numerators and denominators into ascending order**

## **Theory: Sorting multiple ordinary fractions**

## **How to sort multiple fractions?**

* Sorting fractions can be much easier if first the fractions to be sorted are sorted into categories: positive and negative fractions, superunit and subunit fractions.
* As a general rule:
* any positive superunit fraction is greater...
* o ... than any positive equivalent fraction, which is greater ...
* o ... than any positive subunit fraction, which is greater ...
* o ... than zero, which is greater ...
* o ... than any negative subunit fraction, which is greater ...
* o ... than any negative equivalent fraction, which is greater ...
* o ... than any negative superunit fraction.
* If all the fractions are from different categories, then it is very easy to sort, following the above rule.
* If we have more than one fraction in each category, we must first compare the fractions in each category separately, then sort them following the rule above.Mai jos vom sorta în ordine crescătoare trei fracții subunitare pozitive.

### **An example of sorting three positive subunit fractions with different denominators and numerators, with explanations**

1/2 vs. 16/24 vs. 45/75

#### We simplify each fraction separately:

* Decompose the numerator and denominator of each fraction into prime factors;
* It divides the numerator and the denominator by the number obtained by multiplying the common prime factors of the numerator and the denominator, to the lowest powers - this is the greatest common divisor, CMMDC;
* We simplify the fraction 1/2 - the numerator and denominator are coprime numbers, they have no common prime factors, the fraction cannot be simplified, it is irreducible
* We simplify the fraction 16/24 = 24 / (23× 3) = (24 : 23) / ((23× 3) : 23) = 2/3
* We simplify the fraction 45/75 = (32 × 5) / (3 × 52) = ((32 × 5) : (3 × 5)) / ((3 × 52) : (3 × 5)) = 3/5
* At this point, the fractions are simplified:
* 1/2, 16/24 = 2/3 și 45/75 = 3/5

#### We calculate the least common multiple, CMMMC, of ​​the denominators of the new fractions obtained by simplification:

* CMMMC will be the common denominator of the sorted fractions, we can also call it the lowest common denominator.
* We decompose the denominators of the fractions and uniquely choose all the prime factors to the highest powers by multiplying them.
* 2 is a prime number, it can no longer be decomposed into prime factors.
* 3 is a prime number, it can no longer be decomposed into prime factors.
* 5 is a prime number, it can no longer be decomposed into prime factors.CMMMC (2; 3; 5) = 2 × 3 × 5 = 30.

# Sources

<https://mquest.ro/home/learnunitnew?id=32>

<https://mquest.ro/home/ch?c=6>

<https://www.scoalaintuitext.ro/blog/matematica-clasa-a-iii-a-2/>

# Examples



Look at the picture and say who ate less. The 3 slices eaten by Vlad, i.e. 3/8 of the pizza, is less than the 5 slices, i.e. 5/8 that Radu ate.

So,  3/8 **<**5/8.  In this case equal parts of identical wholes were compared.

If the wholes are not the same size, we cannot compare their corresponding fractions. Notice this in the following representation:

Together we discovered that:

Of two fractions with the same denominator, the fraction with the larger numerator is greater.

We can compare two fractions only if they are equal parts of the same whole or equal parts of identical wholes. Rodica helped her grandfather plant vegetables in the garden. The vegetables were distributed according to the following scheme:



We note that:

● on 2/10 of the garden surface they planted beans,

● tomato, on 4/10 of the entire garden,

● 1/10 of the surface is occupied by peppers,

● they planted cabbage on 3/10 of the garden area.

The largest area is cultivated with tomatoes (4/10), and the smallest area with peppers (1/10).

Here is how we sort the fractions corresponding to the areas cultivated with vegetables in ascending order:



# Exercises and problems

1. Write, then compare the represented fractions, using relationship signs (<, >, = ) :



2. Complete the fractions so that the following equations are true:



3. Write a fraction less than and a fraction greater than the given ones:



4. Write all fractions less than or equal to 5/8.

5. Write the fractions represented by coloring in ascending order:



6. Put the fractions between 2/7 and 6/7 in descending orde.

7. Sort fractions with denominator 8 and numerator an odd number less than 6 in ascending order.



