**Logo, company name

Description automatically generated**

**The geometric interpretation of the derivative and derived functions**

**School grade: K12**

**Content**

[GEOMETRIC INTERPRETATION OF THE DERIVATIVE 3](#_Toc125619669)

[DERIVATIVE FUNCTIONS 10](#_Toc125619670)

[The tangent problem to a curve 11](#_Toc125619671)

[Derivability and continuity 13](#_Toc125619672)

[Lateral derivatives 14](#_Toc125619673)

[Derived to the left 14](#_Toc125619674)

[Derived to the right 15](#_Toc125619675)

[Definition of the derivative of a function at a point 15](#_Toc125619676)

[Remarks 16](#_Toc125619677)

[Table with derivatives of elementary functions 17](#_Toc125619678)

[Operations with derivable functions 18](#_Toc125619679)

[Conclusions 19](#_Toc125619680)

[Sources 20](#_Toc125619681)

[Worsksheet 21](#_Toc125619682)

# ****GEOMETRIC INTERPRETATION OF THE DERIVATIVE****

**Diagram, schematic

Description automatically generated**

1. If f’(x0)=∞, then the graph admits a vertical semitangent

Under point M.

2) If f’(x0)=-∞, then the graph admits a vertical semitangent above the point M.

**Diagram, schematic

Description automatically generated**

3) If fd’(x0)=∞, then the graph admits a vertical semitangent above the point M.

**Diagram

Description automatically generated**

4) If fd’(x0)=∞, then the graph admits a vertical semitangent below the point M.

**Diagram

Description automatically generated**

5) If the lateral derivatives are equal fd’(x0)=fs’(x0), then the two tangents are in extension. In this case M is the inflection point (the tangent crosses the graph of the function).

**A picture containing text, music

Description automatically generated**

**A picture containing text, linedrawing

Description automatically generated**

Definition. It is said that x0 is an inflection point of the function f, if the function is continuous at x0, it has a derivative at x0, (finite or infinite)

and if the Graph is conex(concave) on one side of x0 and concave (convex) on the other side.

6) If the lateral derivatives are different and fd’(x0)=+∞,fs’(x0)=- ∞, sau fd’(x0)=-∞,fs’(x0)=+∞, then the two semitangents overlap and M is the turning point.

**Chart

Description automatically generated with low confidence**

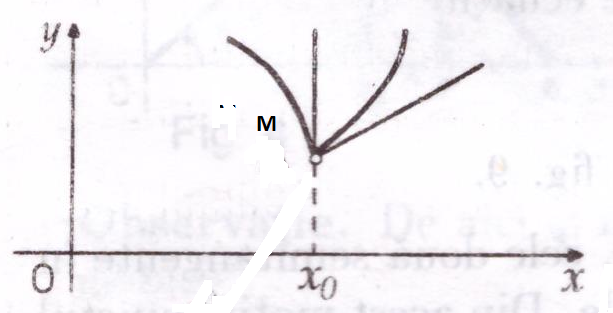
**A picture containing diagram

Description automatically generated**

7) If the lateral derivatives are different and at least one is finite, then M is an angular point.

**Diagram

Description automatically generated with medium confidence**

****

**Case 1) fs’(x0)=-∞, fd’(x0)ϵR**

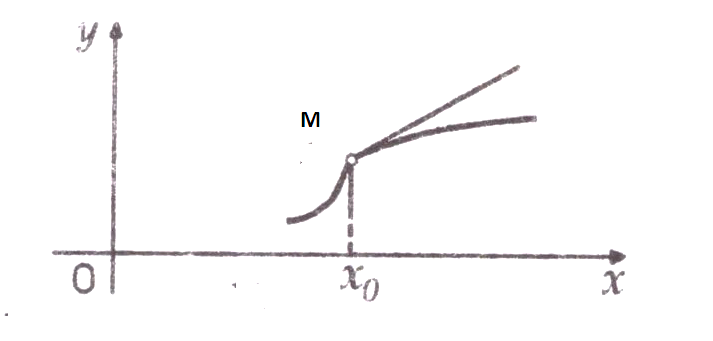
**Diagram

Description automatically generated with medium confidence**

**Case 2) fs’(x0)=+∞, fd’(x0)ϵR**

**Diagram

Description automatically generated**

****

**Diagram

Description automatically generated with medium confidenceA picture containing diagram

Description automatically generated**

**Case 3) fd’(x0)=+∞, fs’(x0)ϵR**

**Case 4) fd’(x0)=-∞, fs’(x0)ϵR**

**A picture containing text

Description automatically generated**

**Diagram

Description automatically generated with medium confidence**

**Case 5) fd’(x0), fs’(x0)ϵR și fd’(x0)≠fs’(x0)**

**A picture containing diagram

Description automatically generated**

# ****DERIVATIVE FUNCTIONS****

**The notion of derivative was introduced and used in mathematics by the scientist Isaac Newton (1642 – 1724) in connection with the study of mechanics.**

**The problem of the instantaneous speed of a mobile**

**the average speed of the mobile in the time interval [t0, t] is**

****

**the instantaneous velocity of the mobile at time t0 (fixed), t0 > 0 is:**

****

**the acceleration of the mobile at the fixed moment t0 is::**

****

**Almost at the same time, the scientist Gottfried Wilhelm Leibniz (1646 – 1716) introduced the notion of derivative in connection with the study of the tangent to a curve at a point of it..**

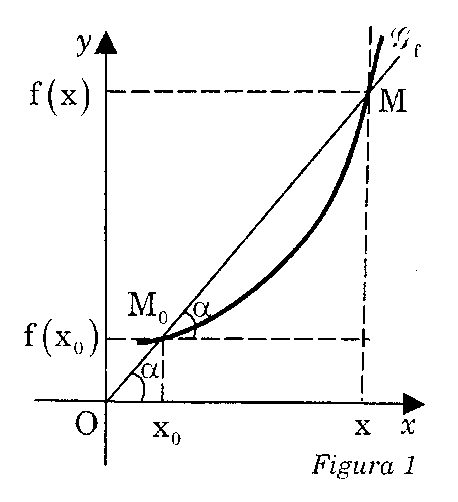
# ****The tangent problem to a curve****

**If f:(a,b)🡪R, a continuous function and M0(x0;f(x0)) on the graphic, Gf on f.**

**The slope of the secant M0M represents the trigonometric tangent of the angle formed by it with the positive direction of the Ox axis.**

****

**The slope or the angular coefficient of the tangent at the point M0 to the curve Gf is:**

****

**The tangent at the point M0(x0,f(x0)) is given by the equation:**

****

****

**The relation (1) is written::**

**and it is called the derivative of the function f at the point x0.**

**Let the function f:D🡪R, D🡪R, x0 Є D an accumulation point of the crowd D.**

**The function f is said to have a derivative at the point x0 Є D if the limit exists:**

****

**This limit is called the derivative of the function f at the point x0, and is written:**

****

**It says that the function f is differentiable at the point x0 Є D if the limit below exists and is finite:**

****

# ****Derivability and continuity****

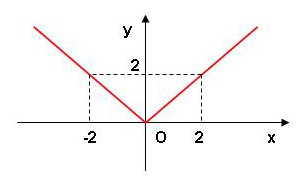
**Any function differentiable at a point is continuous at that point.**

**Remarks:**

**A numerical function can be continuous at a point without being differentiable at that point.**

**Exanple:**

**Mode function f : R🡪R, f(x) =|x| is continuous at x0 = 0 and is not differentiable at the point x0 = 0.**

****

**Any discontinuous function at a point is not differentiable at this point.**

**There are functions that are discontinuous at a point and that have a derivative at that point..**

**Exanple:**

**Function f : R🡪R, given below, is discontinuous in x0 = 0 and f’(0) = + ∞.**

****

# ****Lateral derivatives****

**Let be the function f:D🡪R and x0 Є D.**

# ****Derived to the left****

****

# ****Derived to the right****

****

**The function f has a derivative and is differentiable in x0 if and only if it has lateral derivatives and is, respectively, left and right differentiable in x0 and::**

****

# ****Definition of the derivative of a function at a point****

**Whether f:ER where E is an interval or a union of intervals from R**

**It is said that the function f has a derivative in în if the limit  exists in **

**In this case, this limit is denoted by  and is called the derivative of the function f in **

**So **

**The function f is said to be derived in if the limit  exists in R**

**(exists and is finite)**

**In this case, this limit is denoted by , namely **

**A function f is said to be differentiable on an interval I if it is differentiable at every point of the interval I.**

# ****Remarks****

**The function f has a derivative in x0 f are derivate laterale în x0 and **

**(  exists in  ;  exists in  )**

# ****Table with derivatives of elementary functions****

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **FUNCTION** | **DERIVATIVE** | **FIELD OF DIFFERENTIABILITY** | **COMPOSED FUNCTION** | **DERIVATIVE** |
| **c (constant)** | **0** |  |  |  |
| **x** | **1** |  | **u** |  |
| **x** |  |  |  |  |
| **x**  **( )** |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| **ln x** |  |  | **ln u** |  |
|  |  |  |  |  |
| **sin x** | **cos x** |  | **sin u** | **cos u** |
| **cos x** | **- sin x** |  | **cos u** | **- sin u** |
| **tg x** |  | **cos x** | **tg u (cos u)** |  |
| **ctg x** | **-** | **sin x** | **ctg u (sin u)** |  |
| **arcsin x** |  | **(-1;1)** | **arcsin u** |  |
| **arccos x** | **-** | **(-1;1)** | **arccos u** |  |
| **arctg x** |  |  | **arctg u** |  |
| **arcctg x** | **-** |  | **arcctg u** |  |

# ****Operations with derivable functions****

****

****

****

****

** ( c = constant)**

****

****

****

# ****Conclusions****

**The study of functions in general, of continuous, derivable functions in particular, requires the development of general and specific skills reflected in:**

**Graphical/visual identification of the properties of a numerical function, regarding: boundedness, continuity, asymptotic tendency, derivability;**

**The association of data, extracted from a problem situation, with properties of the numerical functions studied, such as: convergence theorems, limit operations, type limits, derivation tables;**

**The application of specific algorithms, the differential calculation, in solving some problems and modeling some specific processes, some fields of activity;**

**The expression in the language of mathematical analysis, of concrete theorems, which can be modeled by numerical functions;**

**Interpretation, based on graphic reading, of the properties of some functions, which represent examples from the economic, social, scientific field;**

**Experimental verification of the results, deduced by calculation, for practical problems that can be expressed mathematically;**

**Determining some situational optima, by applying differential calculus, in practical or specific problems of some fields of activity.**

**Useful applications of the derivative of a function**

**determining the monotonicity intervals for a given function (is the function increasing or decreasing) – this is done by studying the sign of the first derivative of the function;**

**determining the extreme points for an extended class of numerical functions - this is done by studying the sign of the first derivative of the function;**

**the theoretical results on the monotony and extreme points of a function allow obtaining some inequalities which, with the help of elementary methods, would be difficult to prove;**

**determining the intervals of convexity or concavity of a function - this is done by studying the sign of the second derivative of the function;**

**with the help of derivability it is possible to establish the order of multiplicity of the roots of a polynomial equation or of the intervals in which the roots of an equation associated with a polynomial function are found.**

# ****Sources****

**Gheorghe Cârjă, Ovidiu Cârjă – Analiză matematică, Culegere de probleme rezolvate şi comentate, Editura GIL, Zalău, 2003;**

**Lia Aramă, Toader Morozan – Culegere de probleme de analiză matematică, Editura Universal Pan, Bucureşti, 1997;**

**Marius Burtea, Georgeta Burtea – Matematică, manual pentru clasa a XI-a, Editura Carminis, Piteşti, 2006;**

**Mircea Ganga – Probleme rezolvate din manualele de matematică pentru clasa a XI-a, Editura MATHPRESS, Ploieşti, 2006.**

# Worsksheet

1. Let be f : R → R, f(x)=-3+5
2. Calculate
3. Calculate f’(x)
4. Calculate f’(-1) + f’’(-1)
5. Write the equation of the tangent to the graph of the function f with the abscissa point
6. Calculate
7. Calculate
8. Determine the intervals of monotonicity and the extreme points of the function f.
9. Determine the inflection point of